Evolution of a two-dimensional foam containing a single topological defect: An experimental study

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The evolution of disorder in a two-dimensional foam containing a single topological defect, a large bubble with more than six nearest neighbors, has been studied experimentally. The disorder initially grows, but those bubbles comprising the area of disorder around the central large bubble reach a stationary state in which the distribution of the coordination numbers of the bubbles and the average area per bubble are constant. This affords qualitative support to recent simulations of similar situations in two-dimensional froths; quantitative differences may arise from the wetness of the present foams. [S1063-651X(97)05209-4]

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I. INTRODUCTION

Soap froths are examples of two-dimensional cellular structures whose properties and evolution have been the subject of much attention recently [1–3]. Relatively ordered foam exhibits an initial transient in its evolution [3] that has been interpreted in terms of the growth of disorder from individual topological defects. There is therefore some interest in the study of such growing disorder. In a recent study Levitan investigated the evolution of an isolated defect in an otherwise ideal hexagonal froth [4], concentrating on the cluster of disordered bubbles around the initial defect (the set of bubbles having at least one nonhexagonal neighbor). His results challenged the common wisdom that the scaling state dynamics do not depend on the initial condition, suggesting as they did that the long-time topological distribution function, while of stable form, differed from that for generic initial conditions [random two-dimensional (2D) froth]. This work excited considerable interest and some controversy [5,6], but subsequent computer simulations have suggested that more conventional ideas are more likely correct [7–10].

While there have been experimental studies of the evolution of 2D froths that are initially to a greater or lesser extent disordered [3,11] as well as analogous cellular systems [1], we are unaware of any such studies addressing the specific point at issue in these recent simulations: the behavior of an otherwise ideal 2D system containing one isolated defect. The present paper reports such an experimental study for perfectly sixfold coordinated 2D foam containing a single topological defect. Various types of defect are possible and have been studied in the simulations mentioned above. These include a bound pair of dislocations, formed by performing a single T_1 process at a point within the ideal network [4,8], or a single bubble large enough to have more than six nearest neighbors [7,8]. However, the evolution of the system does not seem to depend critically on the nature of the defect involved [8]. For our experiments we have chosen to use a single bubble that is significantly larger than those forming the body of the foam.

II. EXPERIMENTAL METHODS

The simulations discussed above concern dry 2D froth. While it is possible to make 2D froth that is nearly dry [11], it appears essentially impossible to create perfectly ordered froth. We have adapted Bragg's bubble raft [12] to permit the formation of perfectly ordered 2D foam.

Fortes *et al.* [13] have demonstrated that bubble rafts trapped between the surface of a soap solution and a glass cover plate endure essentially indefinitely, temporal evolution being restricted to that due to coarsening of the bubbles driven by differences in Laplace over-pressure between bubbles of different radii, as for the simulations summarized above. They use such constrained bubble rafts in investigations of the temporal evolution of 2D foams. It proves difficult to avoid polycrystallinity in such experiments, as grain boundaries appear over extended areas, making the creation of large perfect crystals extremely difficult. However, restricting the 2D foam to a hexagonal shape [Fig. 1(a)] helps enforce the desired symmetry [14]. The glass cover is supported just (1–3 mm) above the soap solution on a metal plate that contains a hexagonal hole (typically 6 cm on a side). The plate extends into the solution and the 2D foam is formed by bubbling N_2 into the solution below this hexagonal cell via a long hypodermic needle. The bubbles are attracted to the wall of the cell and to each other by comparatively long-ranged capillary forces [15].

By systematically sweeping the tip of the needle to and fro as the lines of bubbles form it is possible, with practice, to create within the hexagonal cell perfectly sixfold-coordinated lattices comprising several thousand bubbles about 2 mm in diameter. While there must be minor variations in bubble diameter within a given lattice, these are not large enough to affect the regularity of packing to any noticeable degree. The introduction of one or more defects into such 2D foam endows it with a sufficiently long life [14] that it is possible to follow the growth of disorder in the system, permitting experimental tests of the questions raised in computer simulations.

Topological defects, in this case single bigger bubbles having more than six nearest neighbors, are introduced by interrupting the process when the lattice is part made, injecting an isolated bubble of different size using a syringe and hypodermic needle, and then completing the lattice around it. During this process one or two dislocations may form accidentally. However, they can be moved towards each other or towards the big bubble until they disappear, leaving an es-

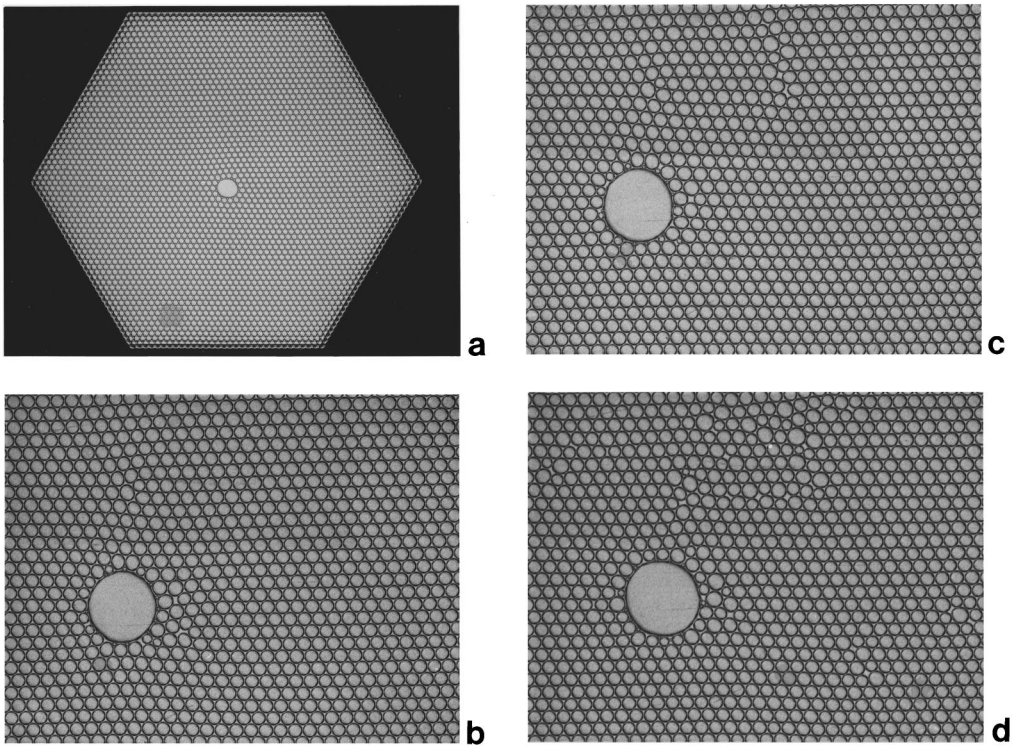

FIG. 1. Pictures of a typical evolving foam that contains an initial isolated larger bubble, constituting a topological defect. (a) The initial 2D foam as formed ($t=0$). (b)–(d) The central portions of the foam after (b) $t=25$ h, (c) $t=28$ h, and (d) $t=32$ h.

sentially ideal hexagonal lattice including a single larger bubble [Fig. 1(a)].

Fortes [16] has pointed out that such large bubbles may have a ''dislocation character,'' in that they have a nonvanishing Burgers vector. Indeed, the example shown in Fig. 1 is of this nature. We find no difference in the temporal evolution of topological defects having a dislocation character and those that lack such a character. The data to be presented below derive from examples of both.

The 2D foams produced as above are, of course, wet, but we may hope that their behavior may reflect at least some of the generic aspects of the evolution of 2D froth as revealed in the simulations [7,8].

III. RESULTS AND DISCUSSION

We present data from a series of experiments with a hexagonal cell 6 cm on a side; the results appear to be independent of system size, at least over a range of cell sizes from 3 to 10 cm. A typical example of the evolution of a single defect in an otherwise regular hexagonal lattice is shown in Fig. 1. The initial cluster [Fig. 1(a)] grows as the disorder increases around the initial defect due to coarsening. Essentially no observable changes occur in the system for 12–15 h, about which time the first increase in disorder around the defect appears. The initial stage of the evolution is faster for larger central bubbles (the number of neighboring bubbles is greater than or equal to 12), as might be expected from von Neumann's law.

We follow Levitan [4] and Jiang *et al.* [7] in studying the evolution of a ''cluster'' defined as all bubbles having at least one neighbor that is not sixfold coordinated. (The alternative definition, excluding the sixfold coordinated bubbles at the cluster periphery [8], leads to larger fluctuations in the distribution of topological classes.) This cluster comprises the large central bubble and a ''boundary'' of disordered bubbles around it. In our experiments, unlike the simulations, inevitable tiny differences in the size of the ''ordered'' bubbles in the body of the foam lead to coarsening of the 2D foam and hence the appearance of generalized disorder in different areas of the foam, over time scales of the order of days [evident in Fig. 1(c)]. This limits the time over which we can follow the evolution of the cluster, as eventually the cluster grows into this coarsening-induced disorder [Fig. 1(d)]. All the data to be presented here relates to times before this occurs.

We have investigated the evolution of certain statistics of the cluster, of the cluster boundary (the cluster minus the large central bubble), and of the central bubble. These include the distribution of so-called topological classes (bubble coordination numbers) and its second moment, as well as the areas and numbers of bubbles involved.

In the simulations time (t) could be used as an independent variable [7,8]. However, in our experiment we follow the time evolution of foams containing topological defects that initially are of different size (quantified here by the number of nearest neighbors, varying from 8 to 16). These different central bubbles do not have the same rate of growth with time, so that it is difficult to compare the different runs as a function of time, as is natural for the simulations. Now von Neumann's law

$$\frac{dA_n}{dt}=\kappa(n-6), \tag{1}$$

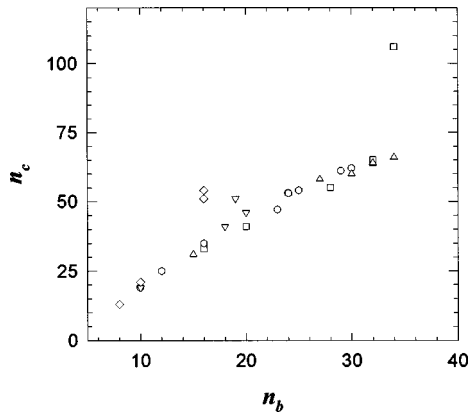

FIG. 2. Number of bubbles comprising the cluster n_c versus the number of neighbors of the topological defect n_b. Here and below different symbols represent data from experiments on different foams containing single large bubbles of different size and hence different n_b.

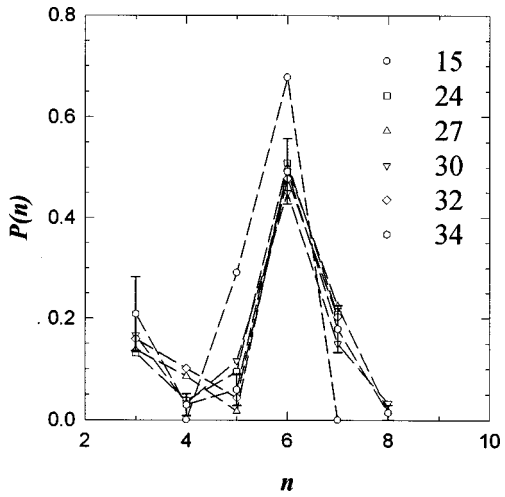

FIG. 3. Topological class distributions for one example of an evolving foam containing an isolated topological defect; values of the number of neighbors of the central bubble (n_b) are shown in the legend. The datum at n_b (corresponding to the large bubble) is omitted. Apart from the distribution for $t=0$ (○) the data for different times are indistinguishable.

where A_n is the size of a bubble with n nearest neighbors and κ is a system-dependent constant, should apply, albeit only statistically for our wet foam. The evolution of the system is governed by that of the large bubble and it seems reasonable to use the number of neighbors of that bubble at a given time (n_b) as the independent variable instead of time itself. We do not claim that n_b depends linearly on t, just that it provides a measure of the temporal evolution of the system. In certain cases it is more natural to use the number of bubbles in the disordered cluster n_c (which was found in the simulations to increase nearly linearly with time).

Experimentally, n_c and n_b both grew with time. The two quantities are related (Fig. 2), supporting the conclusion of the simulations [7] that the outward propagation of the disorder in the foam just follows the growth of the large bubble. This reflects the fact that the boundary is usually only some two bubbles wide (Fig. 1), as found in the simulations [7]. While n_c generally seems to be a smooth function of n_b, some points lie off this variation: towards the end of an experiment, n_c sometimes increases relative to the general trend of the dependence upon n_b. It may be that this occurs as the cluster approaches the regions of coarsening-induced disorder, although we cannot presently confirm this. This feature does not seem to perturb any of the data presented below.

The tail of the topological distribution function $P(n)$ extended towards larger values of n as time progressed, while the peak of the distribution stayed at $n=6$ due to the definition of the cluster as including an outer ring of sixfold-coordinated bubbles with at least one neighbor with $n\neq 6$. Figure 3 shows typical topological distributions for those bubbles belonging to the boundary (i.e., omitting the datum at n_b, the number of neighbors of the large central bubble). The error bars on all these distributions are approximately the same and for clarity are shown only for one case.

An unusual feature of these distributions is the significant population at $n=3$; in conventional 2D froths such bubbles disappear through the T_2 process [1]. The threefold-coordinated bubbles are small and are observed to lie around the central large bubble, adjacent to two more normal sized bubbles; examples are apparent in Figs. 1(b)–1(d). The difference from 2D froths clearly lies in the wetness of our foams. The area of contact available for diffusive interchange of N_2 between the smallest bubbles and their neighbors falls more rapidly for the present quasispherical bubbles than in the usual 2D froths, hindering the final stages of evolution leading to the T_2 process. We should note another complication of our wet foams: the Plateau borders between the bubbles lose their triangular shape, multiple borders forming as threefold borders merge [17]. Such borders potentially lead to some ambiguity concerning adjacency of bubbles, but in practice this can always be resolved unambiguously.

$P(n)$ changes from its initial form (the boundary of the cluster comprising only fivefold- or sixfold-coordinated bubbles) as the cluster becomes more disordered. However, apart from the point at n_b, $P(n)$ quite rapidly reaches a stationary form; within the uncertainties the distributions shown in Fig. 3 for $n_b \geq 24$ ($t>10$ h) are indistinguishable. Turning to quantitative measures, the second moment of the topological class distribution of the cluster (μ_2) grows monotonically with n_b [Fig. 4(a)], reaching rather large values. The experimental cutoff imposed as the cluster runs into coarsening-induced disorder in the initially ordered part of the foam prevents these diverging values of μ_2 from reaching the values attained in simulations [7,8]. However, the continuing increase in μ_2 is essentially entirely due to the increase of n_b, the coordination number of the central large bubble. The second moment of the boundary μ_2', which excludes this value, appears to saturate at a value about 2 [Fig. 4(b)]. Apparent fluctuations in the initial behavior of μ_2' are due to the different rates of growth for central bubbles having different initial values of n_b. In general, μ_2' starts at a very low value, about 0.2, and quite rapidly makes a transition to the regime of apparent saturation, intermediate values only occasionally being observed.

Thus $P(n)$, omitting n_b, and the corresponding second moment μ_2' become constant early in the evolution of the

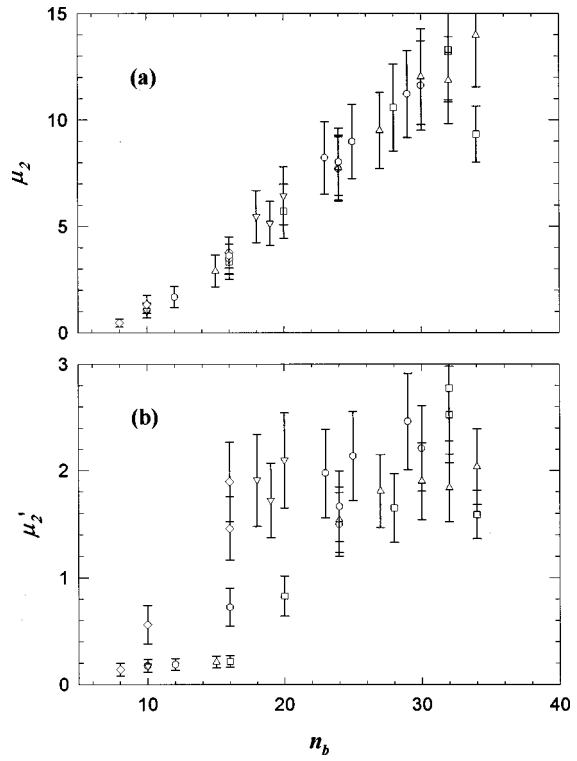

FIG. 4. (a) Variation of μ_2, the second moment of the cluster, with n_b as different foams evolve. (b) Evolution of μ_2', the second moment of the boundary.

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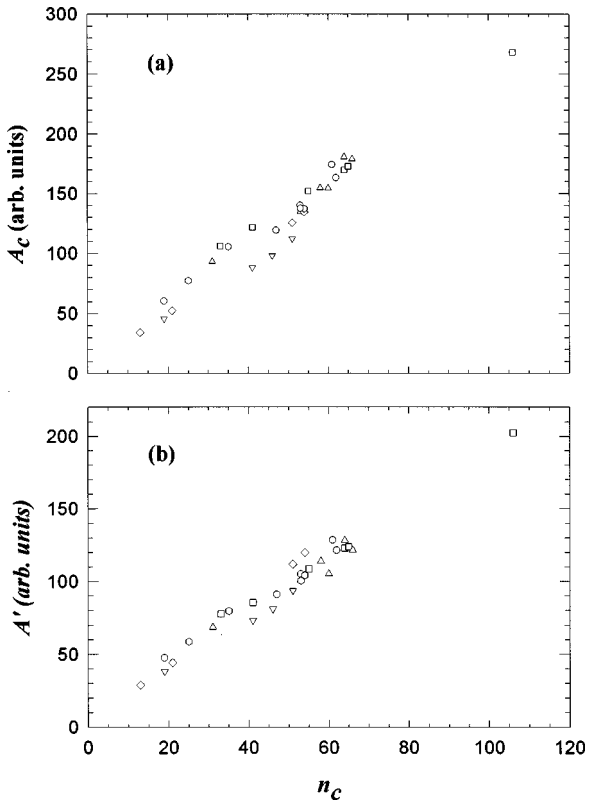

FIG. 5. Variation of the area of (a) the cluster (A_c) and (b) the boundary (A') with n_c as different foams evolve.

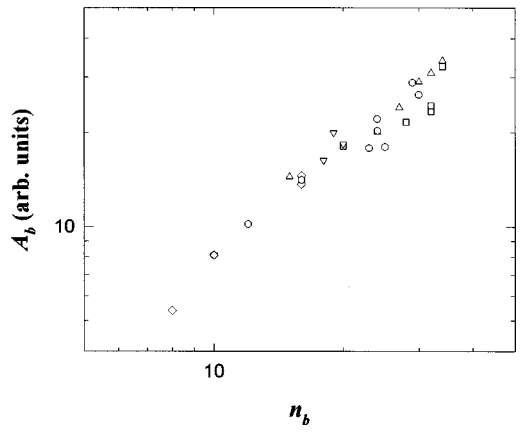

FIG. 6. Dependence of the area of the central bubble on n_b as the foam evolves.

system, suggesting that the boundary of the cluster quite rapidly reaches a scaling state. While this supports conclusions drawn from simulations [7], the weighted mean of μ_2' following saturation, 1.9 ± 0.1, is considerably higher than the corresponding value from the simulations (0.71 ± 0.17). However, comparable values of μ_2 have been observed in the initial transient behavior of initially relatively ordered 2D froths [3]. While its value does fluctuate somewhat, μ_2' in the simulations [7] seems to grow slowly with time. The high-μ_2' data of Fig. 4(b) may be constant or may increase gently as n_b grows (statistical tests fail to discriminate between these possibilities). Unfortunately, it will not be easy to pursue this variation to higher n_b to check this point because of the problem of the coarsening-induced disorder elsewhere in the foam discussed above. Such longer time studies would also be interesting regarding the question whether the present large values of μ_2' are merely a transient phenomenon, as seen in studies of 2D froths [3].

We turn now to the areas of the cluster, the cluster boundary, and the center bubble. The areas of the cluster A_c and of the cluster boundary A' increase roughly linearly with the number of bubbles in the cluster n_c (Fig. 5), as might be expected. However, the area of the central bubble (A_b) does not behave as expected (Fig. 6). It seems plausible that we should expect $A_b \propto n_b^2$, as the circumference of the growing bubble can accommodate more neighboring bubbles. Indeed, in the simulations it is found that $A_b \propto t^2$, n_b rising roughly

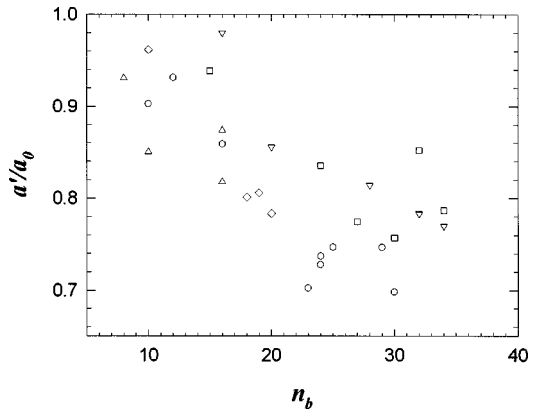

FIG. 7. Variation of the normalized area per bubble in the boundary as the foam evolves. After an initial decline, the data appear to fluctuate about a stable value.

linearly with t [7,8,10]. The departure of A_b from the expected n_b^2 dependence implies that the area of the neighboring bubbles falls with time (increasing n_b): n_b is larger than one would anticipate for a given A_b. Indeed, the dependence of A' upon n_c [Fig. 5(b)] supports this suggestion. The average area per bubble in the boundary (a'), when normalized by that measured in an ordered region of the foam at the same age (a_0), falls with time, but ultimately fluctuates about a constant value for $n_b \gtrsim 20$ (Fig. 7). This normalized area per bubble includes the Plateau borders between the bubbles, which are not insignificant in our wet foam. In the scaling state for $n_b > 20$ (corresponding to the regime where μ_2' is constant) the average value of a'/a_0 is 0.77 ± 0.04. While this is quite close to values found in simulations (0.88 ± 0.08 [7]), we emphasize that our value need *not* be comparable: in the simulations it is the average area of a bubble in the boundary that is determined, whereas we measure the average area per bubble.

IV. CONCLUSION

The main purpose of this work was to investigate experimentally the evolution of a single defect in an isotropic hexagonal lattice for comparison with recent computer simulations [7,8]. Central bubbles with different numbers of neighbors were studied. In general terms the results afford qualitative support for the conclusion from simulations that the area of disorder that evolves around the initial defect (referred to above as the boundary) achieves a scaling state. This boundary comprises different bubbles at different times; in it the distribution of topological classes reaches a stationary form having constant second moment μ_2' and the area per bubble becomes constant. Certain quantitative differences of detail between the present data and the simulation results appear to originate in the wetness of the foam studied. Further simulations of the evolution of disorder around a single defect in an otherwise ordered foam incorporating different liquid fractions would be useful in pursuing these differences.

ACKNOWLEDGMENTS

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